

Publication

June 2005
6685 Statistics S3
Mark Scheme

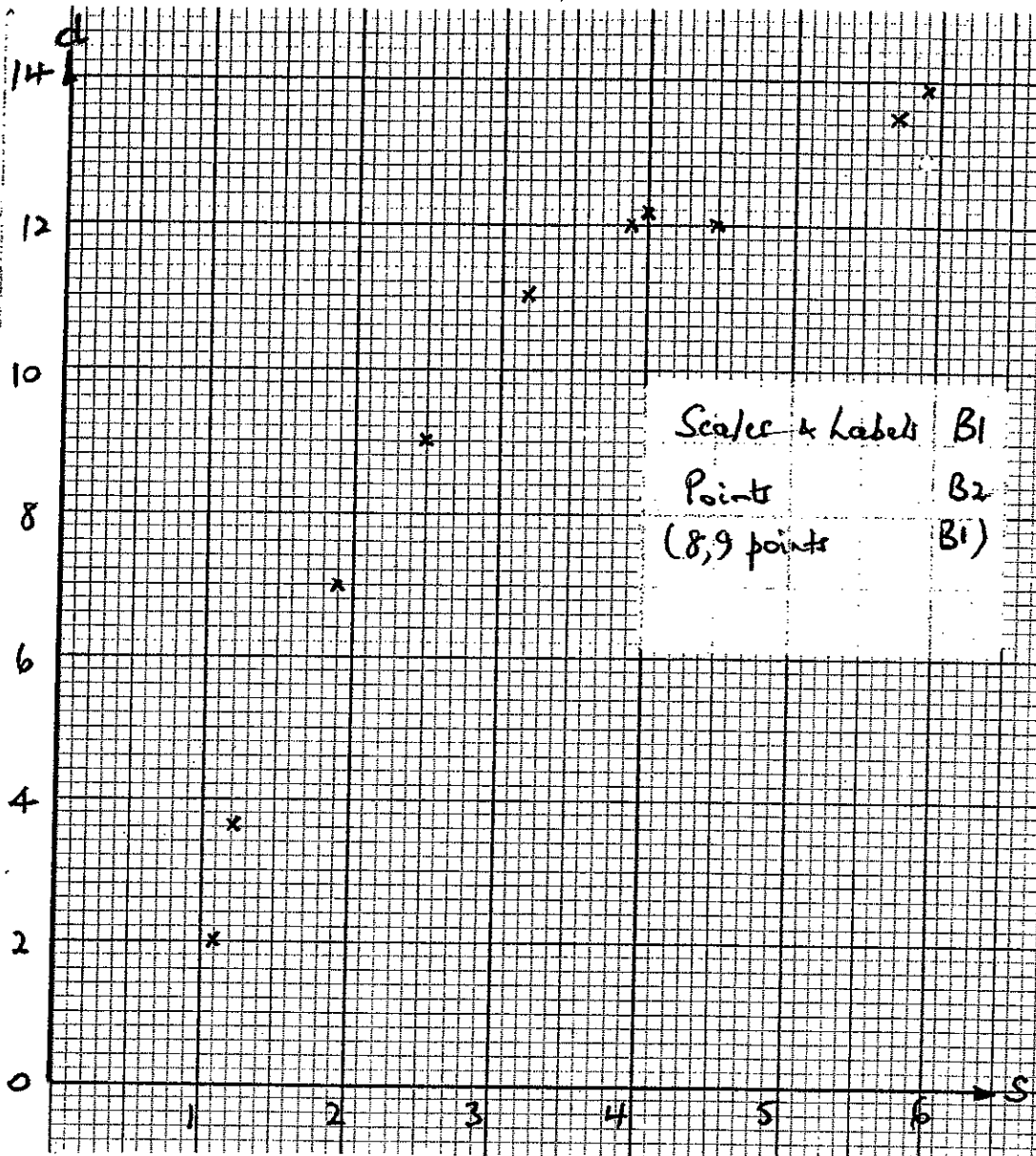
FINAL

| Question Number | Scheme | Marks |
|-----------------|--|--|
| 1. | <p>(a) Population divides into <u>mutually exclusive</u>; groups <u>distinct</u> strata</p> <p>(b) <u>Advantages</u></p> <ul style="list-style-type: none"> - enables fieldwork to be done quickly - costs kept to a minimum - administration is relatively easy <p><u>Disadvantages</u></p> <ul style="list-style-type: none"> - non-random so not possible to estimate sampling error - Subject to possible interviewer bias - non-response not recorded | <p>B1; B1 (2)</p> <p>Any ONE B1</p> <p>Any ONE B1 (2)</p> |
| 2. | <p>$X \sim N(10, 3^2) \therefore \bar{X} \sim N(10, 9/5)$ can be implied 10; 9/5</p> <p>$P(7 \leq \bar{X} \leq 10) = P\left(\frac{7-10}{\sqrt{9/5}} < Z < 0\right)$ Standardising with 10 & their σ</p> <p>$= P(-2.236 < Z < 0)$ 2.236</p> <p>$= \Phi(0) - [1 - \Phi(2.24)]$</p> <p>$= \underline{0.4875}$</p> | <p>B1; B1</p> <p>M1 A1</p> <p>A1</p> <p>M1 (p < 0.5)</p> <p>A1 (6)</p> |

| Question Number | Scheme | | | | Marks | | | | | | | | | | | | | | | | | | | | | | | | | |
|--|--|--------------------------|----------------------|-------|---|-----------|--------------------------|----------------------|-------|-------------------------|--------|-------|-------|----|--------------------|-------|-------|-------|----|--------------------|-------|-------|-------|----|--------|----|----|----|----|--|
| 3. | <table border="1"> <thead> <tr> <th></th> <th>No action</th> <th>Remove diseased branches</th> <th>Spray with Chemicals</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Tree died within 1 year</td> <td>10 (7)</td> <td>5 (7)</td> <td>6 (7)</td> <td>21</td> </tr> <tr> <td>Survived 1-4 years</td> <td>5 (7)</td> <td>9 (7)</td> <td>7 (7)</td> <td>21</td> </tr> <tr> <td>Survived > 4 years</td> <td>5 (6)</td> <td>6 (6)</td> <td>7 (6)</td> <td>18</td> </tr> <tr> <td>Totals</td> <td>20</td> <td>20</td> <td>20</td> <td>60</td> </tr> </tbody> </table> | | | | | No action | Remove diseased branches | Spray with Chemicals | Total | Tree died within 1 year | 10 (7) | 5 (7) | 6 (7) | 21 | Survived 1-4 years | 5 (7) | 9 (7) | 7 (7) | 21 | Survived > 4 years | 5 (6) | 6 (6) | 7 (6) | 18 | Totals | 20 | 20 | 20 | 60 | |
| | No action | Remove diseased branches | Spray with Chemicals | Total | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Tree died within 1 year | 10 (7) | 5 (7) | 6 (7) | 21 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Survived 1-4 years | 5 (7) | 9 (7) | 7 (7) | 21 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Survived > 4 years | 5 (6) | 6 (6) | 7 (6) | 18 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Totals | 20 | 20 | 20 | 60 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| $\frac{RT \times CT}{GT}$ $\frac{6 \times 7}{3 \times 6}$ | | | | | M1 A1 A1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| H_0 : Treatment & Survival are independent (not associated) H_1 : Treatment & Survival are not independent (associated) | | | | | BI both | | | | | | | | | | | | | | | | | | | | | | | | | |
| $\alpha = 0.05$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| $L = (3-1) \times (3-1) = 4$ | | | | | BI | | | | | | | | | | | | | | | | | | | | | | | | | |
| $CR: \chi^2 > 9.488$ | | | | | BI ✓ | | | | | | | | | | | | | | | | | | | | | | | | | |
| $\sum \frac{(O-E)^2}{E} = \frac{9}{7} + \frac{4}{7} + \frac{1}{7} + \frac{4}{7} + \frac{4}{7} + 0 + \frac{1}{6} + 0 + \frac{1}{6}$ $= 3.47619 \dots$ | | | | | M1 A1 A1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Since 3.47619... is NOT in the critical region (ie < 9.488) there is insufficient evidence to reject H_0.</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>There is no evidence of association between treatment and length of survival.</p> | | | | | Comparison Conclusion M1 A1 ✓ (11) | | | | | | | | | | | | | | | | | | | | | | | | | |

4

(a)

NB No graph paper \Rightarrow 0/3

(3)

(b) linear association between s and d

B1 (1)

$$(c) S_{ss} = 141.51 - \frac{33.9^2}{10} = 26.589; S_{dd} = 152.444; S_{sd} = 59.524$$

B1; B1; B1 (3)

$$(d) r = \frac{59.524}{\sqrt{152.444 \times 26.589}}$$

M1

$$= 0.93494\dots$$

AWRT 0.935

A1 (2)

(e) $H_0: \rho = 0$; $H_1: \rho > 0$

Critical Value at 1% = 0.7155

Reject H_0 ; Levels of serum & disease are positively correlated

(f) linear correlation significant ~~but~~ ^{but} scatter diagram looks non-linear.

B1

B1

B1 (3)

B1 (1)

5.

H_0 : Poisson distribution is a suitable model both

H_1 : Poisson distribution is not a suitable model

$$\hat{\lambda} = \frac{(0 \times 99) + (1 \times 65) + \dots + (4 \times 2)}{200} = \frac{153}{200} = 0.765$$

M1 A1

Using $P(X=x) = \frac{0.765^x e^{-0.765}}{x!}$ where X represents the number of restarts given $200 \times P(X=x)$

M1

| X | Observed Frequency | Expected Frequency | |
|----------|--------------------|--------------------|-----------|
| 0 | 99 | 93.06678... | |
| 1 | 65 | 71.19604... | 0, 1, 2 |
| 2 | 22 | 27.23250... | |
| 3 | 12 | 6.94428... | } 8.50468 |
| ≥ 4 | 2 | 1.56040... | |

A1, A1
(-1e.e.)

A1

$\chi^2 = 4 - 1 - 1 = 2$; CR: $\chi^2 > 5.991$ from Poisson
 $\chi^2 = 4 - 1 = 3$; CR: $\chi^2 > 7.815$ from Poisson (0.765)
 $\sum \frac{(O-E)^2}{E} = 5.47368...$ OR $\sum \frac{(O-E)^2}{E}$

B1; B1✓

M1

A1

5.47 is not in the critical region.

~~5.40 - 5.50~~

Number of computer failures per day can be modelled by a Poisson distribution

A1✓ (12)

| | | |
|----|---|---|
| 6. | <p>(a) let X represent repair time</p> <p>$\therefore \sum x = 1435 \quad \therefore \bar{x} = \frac{1435}{5} = \underline{287}$</p> <p>$\sum x^2 = 442575 \quad \therefore s^2 = \frac{1}{4} \left\{ 442575 - \frac{1435^2}{5} \right\}$</p> <p style="text-align: center;">$= \underline{7682.5}$</p> <p>(b) $P(\mu - \bar{x} < 20) = 0.95$</p> <p style="text-align: right; font-size: small;">Use of 1.96 for 95% with their σ & \sqrt{n} 1.96</p> <p>$\therefore \frac{20}{\sigma/\sqrt{n}} = 1.96$</p> <p>$\therefore n = \frac{1.96^2 \sigma^2}{20^2} = \frac{1.96^2 \times 100^2}{400} = \underline{96.04}$</p> <p style="text-align: right; font-size: small;">Solving for n</p> <p>\therefore <u>Sample size $(\geq) 97$ required</u></p> | <p>BI</p> <p>MIAI</p> <p>AI (4)</p> <p>MIAI MI</p> <p>BI BI</p> <p>MIAI AI</p> <p>MI</p> <p>AI</p> <p>AI (6)</p> |
| 7. | <p>let $W = C_1 - C_2$ NB $W = C_1 + C_2 \Rightarrow$ MIA or MI only</p> <p>(a) $\therefore W \sim N(0, 16)$ Normal $0; 16$</p> <p>$\therefore P(W > 6) = 2P(W > 6)$</p> <p>$= 2 \times P\left(Z > \frac{6-0}{\sqrt{16}}\right)$ Standardizing, their σ</p> <p>$= 2 \times P(Z > 1.5)$</p> <p>$= 2 \times (1 - 0.9332) = \underline{0.1336}$</p> <p>NB $W = C - L$ treat as MR Prob = 0.4346</p> <p>(b) Let $W = C - L$</p> <p>$\therefore W \sim N(5, 25)$ $5; 25$</p> <p>$P(W > 0) = P\left(Z > \frac{0-5}{\sqrt{25}}\right)$</p> <p>$= P(Z < 1)$</p> <p>$= \underline{0.8413}$</p> | <p>MI</p> <p>AI; MI</p> <p>MI</p> <p>MI</p> <p>AI (6)</p> <p>BI; BI</p> <p>MIAI</p> <p>MI ($p > 0.5$)</p> <p>AI (6)</p> |

(g) Let $W = C_1 + \dots + C_{24} + B$

$\therefore E(W) = 24 \times 350 + 100 = \underline{8500}$

$Var(W) = 24 \times 8 + 2^2 = \underline{196}$

$P(8510 \leq W \leq 8520) = P\left(\frac{8510 - 8500}{\sqrt{196}} \leq Z \leq \frac{8520 - 8500}{\sqrt{196}}\right)$

$= P(0.714 \leq Z \leq 1.428)$ AwRT

$= 0.9236 - 0.7611$

$= \underline{0.1625}$

$0.161 - 0.163$

BI

BI

MI

AI/ AI/

AI (6)

BI (1)

(d) All random variables are independent.

Y.E. Sripuneth
13/06/05

June 2006
6691 Statistics S3
Mark Scheme

| Question Number | Scheme | Marks |
|-----------------|---|---|
| 1 (a) | <p><u>Advantages:</u></p> <ul style="list-style-type: none"> - does not require the existence of a ^{Sampling frame} population list - <u>field work can be done quickly</u> as representative sample can be achieved with a small sample size - costs kept to a minimum (<u>cheaply</u>) - administration relatively <u>easy</u> - non-response not an issue <p><u>Disadvantages:</u></p> <ul style="list-style-type: none"> - not possible to estimate sampling errors - interviewer choice and may not be able to judge easily / <u>may lead to bias</u> - non-response not recorded - non-random process | <p style="text-align: right;">any one B1</p> <p style="text-align: right;">any one B1</p> <p style="text-align: right;">(2)</p> |
| (b) | <p><u>Advantages:</u></p> <ul style="list-style-type: none"> - <u>random process</u> so possible to <u>estimate sampling errors</u> - free from <u>bias</u> <p><u>Disadvantages:</u></p> <ul style="list-style-type: none"> - not suitable when sample size is large - <u>sampling frame required</u> which <u>may not exist</u> or may be difficult to construct for a large population. | <p style="text-align: right;">any one B1</p> <p style="text-align: right;">any one B1 (2)</p> <p style="text-align: right;">TOTAL 4</p> |

NO REPETITION / OPPOSITES

| Question Number | Scheme | Marks |
|-----------------|---|---|
| 2 (a) | $\bar{X} \sim N(90, \frac{\Sigma^2}{100}) \text{ i.e. } N_9(90, 0.25)$ <p>Application of <u>central limit theorem</u> as (sample large)</p> | M1A1 B1 (3) |
| 2 (b) | $P(\bar{X} \geq 91) = 1 - P(Z < \frac{91-90}{0.5}) \quad \text{stand.}$ $= 1 - P(Z < 2)$ $= 1 - 0.9772$ $= 0.0228 \quad \text{aurl } 0.0228$ | M1A1 A1 (3) TOTAL 6 |
| 3 (a) | $H_0: \mu_A = \mu_B, H_1: \mu_A \neq \mu_B \quad \mu_1, \mu_2 \text{ OK both}$ $s_e = \sqrt{\frac{47^2}{70} + \frac{23^2}{90}} (= \sqrt{37.43492...})$ <p>Test statistic is $\pm \frac{198-201}{s_e} = \pm 0.4903 \quad \text{aurl } 0.99$ <small>M1A1 probab aurl 0.312 B1 probab cv 0.025</small></p> $cv = (\pm) 1.96$ <p>Insufficient evidence to reject H_0, no significant difference between the mean cholesterol content of the two samples. (require correct comparison for FT) <u>content required.</u></p> | B1 M1A1 M1A1 B1 A1 ✓ (7) |
| 3 (b) | <ul style="list-style-type: none"> - require 1 egg from each of 70 chickens of diet A to ensure <u>independence</u>, similarly for diet B. - no chickens in common between the two samples to ensure <u>independence</u> - not same chickens on diet A and diet B because if it were we need to do a <u>paired analysis</u>. <p style="text-align: right;">Any 1</p> <p>not same chickens on diet A and diet B because if it were we need to do a paired analysis.</p> | B1, B1 (2) TOTAL 9 |

4.

Rank:

| Shop | Distance | Price | d | d ² |
|------|----------|-------|---|----------------|
| A | 1 | 9 | 8 | 64 |
| B | 2 | 7 | 5 | 25 |
| C | 3 | 10 | 7 | 49 |
| D | 4 | 6 | 2 | 4 |
| E | 5 | 4 | 1 | 1 |
| F | 6 | 8 | 2 | 4 |
| G | 7 | 2 | 5 | 25 |
| H | 8 | 1 | 7 | 49 |
| I | 9 | 5 | 4 | 16 |
| J | 10 | 3 | 7 | 49 |

Reverse ranking on price, $\sum d^2 = 44$
Hairs

(a)

$$r_s = 1 - \frac{6 \times 286}{10(100-1)} = -0.73 \text{ or } \frac{-11}{15} \text{ or } -0.733$$

(5)
or 0.733 for $\sum d^2 = 44$

(b)

$H_0: \rho = 0$

$H_1: \rho < 0$

cv = -0.5636

(H₁: $\rho > 0$ if reverse ranking)

(0.5636)

Reject H₀, evidence there is a significant
negative correlation between the price of an
ice cream and the distance from a tourist attraction.

(Ice cream gets cheaper further from the tourist attraction)

(-cv from correct table required) (positive in context)

M1

M1, A1

M1 A1

(5)

B1

B1

B1

B1

(4)

TOTAL 9

5.

 $M = \text{wt of male worker}$

$M \sim N(78.5, 12.6^2)$

 $F = \text{wt of female worker}$

$F \sim N(62.0, 9.8^2)$

(a) $W = M_1 + \dots + M_7 + F_1 + \dots + F_8$

$E(W) = 7 \times 78.5 + 8 \times 62.0 = 1045.50$

awrt
1050 M1A1

$\text{Var}(W) = 7 \times 12.6^2 + 8 \times 9.8^2 = 1879.64$

1880 M1A1

(4)

(b) Independent: (used in Variance formula)

B1

(1)

(c) $P(W > 1090) = P\left(Z > \frac{1090 - 1045.5}{\sqrt{1879.64}}\right)$

M1

$= P(Z > 1.03)$

awrt 1.03
A1

$= 1 - 0.8485$

1 - H1

$= \underline{0.1515}$

A1

(4)

AWRT(0.152)

(9)

| 6. | <p>H_0: No association between age and colour (independent)</p> <p>H_1: Association between age and colour (Not independent)</p> <table border="1" data-bbox="383 481 989 918"> <thead> <tr> <th>O</th> <th>E</th> <th>$\frac{(O-E)^2}{E}$</th> </tr> </thead> <tbody> <tr> <td>12</td> <td>10.08</td> <td>0.3657...</td> </tr> <tr> <td>6</td> <td>7.92</td> <td>0.4654...</td> </tr> <tr> <td>10</td> <td>9.52</td> <td>0.0242...</td> </tr> <tr> <td>7</td> <td>7.48</td> <td>0.0308...</td> </tr> <tr> <td>6</td> <td>8.4</td> <td>0.6857...</td> </tr> <tr> <td>9</td> <td>6.6</td> <td>0.8727...</td> </tr> </tbody> </table> <p>$\sum \frac{(O-E)^2}{E} = 2.4446...$</p> <p>$\nu = (3-1)(2-1) = 2, \chi^2 = 5.991$</p> <p>Insufficient evidence to reject H_0.</p> <p>No association between age and colour</p> <p>(cv for correct h/c for ft)</p> | O | E | $\frac{(O-E)^2}{E}$ | 12 | 10.08 | 0.3657... | 6 | 7.92 | 0.4654... | 10 | 9.52 | 0.0242... | 7 | 7.48 | 0.0308... | 6 | 8.4 | 0.6857... | 9 | 6.6 | 0.8727... | <p>BI</p> <p>BI</p> <p>MIAI</p> <p>MIAI</p> <p>MIAI</p> <p>BI BI ✓</p> <p>A I ✓ (ii)</p> <p>TOTAL 11</p> |
|-------|---|---|---|---------------------|----|-------|-----------|---|------|-----------|----|------|-----------|---|------|-----------|---|-----|-----------|---|-----|-----------|--|
| O | E | $\frac{(O-E)^2}{E}$ | | | | | | | | | | | | | | | | | | | | | |
| 12 | 10.08 | 0.3657... | | | | | | | | | | | | | | | | | | | | | |
| 6 | 7.92 | 0.4654... | | | | | | | | | | | | | | | | | | | | | |
| 10 | 9.52 | 0.0242... | | | | | | | | | | | | | | | | | | | | | |
| 7 | 7.48 | 0.0308... | | | | | | | | | | | | | | | | | | | | | |
| 6 | 8.4 | 0.6857... | | | | | | | | | | | | | | | | | | | | | |
| 9 | 6.6 | 0.8727... | | | | | | | | | | | | | | | | | | | | | |
| 7.(a) | <p>$\bar{x} = \frac{500}{10} = 50$</p> <p>$s^2 = \frac{1}{9} (25001.74 - \frac{500^2}{10}) = 0.193$</p> <p>limits are 50 ± 1.966</p> <p>$= (49.02, 50.98)$</p> <p>Confidence interval is</p> <p>$(50 - 2.5758 \times \frac{0.5}{\sqrt{10}}, 50 + 2.5758 \times \frac{0.5}{\sqrt{10}})$</p> <p>$= (49.59273, 50.40727...)$</p> <p>use of estimate in (a) in (b) AND (c) assume MISREAD.</p> | <p>MIAI</p> <p>MIAIAI (5)</p> <p>MIBI</p> <p>AIAI (4)</p> <p>MIBIAN</p> <p>AIAI (5)</p> <p>TOTAL 14</p> | | | | | | | | | | | | | | | | | | | | | |

8 (a)

$B_7(5, 0.5)$

MIAI
(2)

(b)

H_0 : $B(5, 0.5)$ is a suitable model (good fit)

H_1 : $B(5, 0.5)$ is not a suitable model (not a good fit)
✓ for $\hat{p} = 0.466$.

BI ✓

| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 |
|--------------|-------|--------|-------|-------|--------|-------|
| Expected | 3.125 | 15.625 | 31.25 | 31.25 | 15.625 | 3.125 |
| Actual | 6 | 18 | 29 | 34 | 10 | 3 |

100% (100%)
For Bin,
1 correct = AI
All correct = AI
3st or better

MIAIAI

| | O | E | $\frac{(O-E)^2}{E}$ |
|--------|----|-------|---------------------|
| 0 or 1 | 24 | 18.75 | 1.47 |
| 2 | 29 | 31.25 | 0.162 |
| 3 | 34 | 31.25 | 0.242 |
| 4 or 5 | 13 | 18.75 | 1.763 |

grouped O and E
All count 2st or better.

MIAI

$$\sum \frac{(O-E)^2}{E} = 3.6373$$

Σ required, count 3.64

MIAI

$$\nu = 4 - 1 = 3, \chi^2_{0.10}(3) = 6.251$$

BI BI ✓

Insufficient evidence to reject H_0
 $B(5, 0.5)$ is a suitable model.

No evidence that coins are biased

AI ✓

(11)

Ungrouped gives count 5.44, $\nu = 5, \chi^2_5 = 9.236$
~~for 100% correct~~

TOTAL 13

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Statistics S3 (6691)

June 2007
6691 Statistics S3
Mark Scheme

| Question number | Scheme | Marks | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--------------------------|--|---|---|---|---|---|---|---|---|---|---------------|---|---|---|---|---|---|---|---|---------------|---|---|---|---|---|---|---|---|-------|---|---|---|---|---|---|---|---|---|
| <p>1. (a)</p> | <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> </tr> </thead> <tbody> <tr> <td><i>P</i> Rank</td> <td>2</td> <td>6</td> <td>4</td> <td>3</td> <td>1</td> <td>7</td> <td>8</td> <td>5</td> </tr> <tr> <td><i>Q</i> Rank</td> <td>2</td> <td>8</td> <td>1</td> <td>6</td> <td>3</td> <td>5</td> <td>7</td> <td>4</td> </tr> <tr> <td>d^2</td> <td>0</td> <td>4</td> <td>9</td> <td>9</td> <td>4</td> <td>4</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p style="text-align: right; margin-right: 20px;">$\sum d^2 = 32$</p> $r_s = 1 - \frac{6 \times 32}{8 \times (8^2 - 1)}$ $= \frac{13}{21} \text{ or AWRT } 0.619$ | | A | B | C | D | E | F | G | H | <i>P</i> Rank | 2 | 6 | 4 | 3 | 1 | 7 | 8 | 5 | <i>Q</i> Rank | 2 | 8 | 1 | 6 | 3 | 5 | 7 | 4 | d^2 | 0 | 4 | 9 | 9 | 4 | 4 | 1 | 1 | <p>M1A1</p> <p>M1A1</p> <p>M1</p> <p>A1 (6)</p> |
| | A | B | C | D | E | F | G | H | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <i>P</i> Rank | 2 | 6 | 4 | 3 | 1 | 7 | 8 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <i>Q</i> Rank | 2 | 8 | 1 | 6 | 3 | 5 | 7 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| d^2 | 0 | 4 | 9 | 9 | 4 | 4 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>(b)</p> | <p>$H_0 : \rho = 0$ $H_1 : \rho > 0$ (ρ_s is OK) both</p> <p>r_s 1 tail 5% critical value is 0.6429 (Independent of their H_1)</p> <p>$0.619 < 0.6429$ or not significant</p> <p>So insufficient evidence of a positive correlation between judges competitor <u>is</u> justified</p> <p><u>Or</u></p> | <p>B1</p> <p>B1 (\pm is OK)</p> <p>M1</p> <p>A1f.t. (4)</p> <p style="text-align: center;">10</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>(a)</p> | <p>1st M1 for attempting to rank both <i>P</i> and <i>Q</i>. 1st A1 for both correct (could be reversed) 2nd M1 for attempting d^2 2nd A1 for $\sum d^2 = 32$. 3rd M1 for correct use of formula for r_s</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>(b)</p> | <p>M1 for a correct comparison or statement about significance (o.e.) Follow through their r_s provided $0 < r_s < 1$</p> <p>A1f.t. for a conclusion in context. Must mention judges or marks or competitor. If they use correlation they must say it is positive. Follow through their positive r_s with their positive c.v. and ignore hypotheses. So $r_s = 0.667$ they could say competitor's claim is not justified etc.</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>S.C.</p> | <p><u>No ranking</u> Typical answer (-3.82) can get mark for use of r_s formula and hypotheses in (b) only</p> <p>(a) M0A0M0A0M1A0 (b) B1B1M0A0</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question number | Scheme | Marks | | | | | | |
|-----------------|--|-------|------|------|----|------|------|---|
| <p>2. (a)</p> | <p>H_0 : Maths grades are independent of English grades <u>or</u> No association ... H_1 : Maths and English grades are dependent <u>or</u> There is an association ...</p> <p>Expected Frequencies e.g. $\frac{60 \times 40}{120} = 20$</p> <table border="1" data-bbox="826 434 1072 510"> <tr> <td>20</td> <td>27.5</td> <td>12.5</td> </tr> <tr> <td>20</td> <td>27.5</td> <td>12.5</td> </tr> </table> $\sum \frac{(O-E)^2}{E} = 2 \times \left(\frac{5^2}{20} + \frac{2.5^2}{27.5} + \frac{2.5^2}{12.5} \right), = 3.9545\dots \quad \text{AWRT } \underline{3.95} \text{ or } \underline{3.955}$ <p>$\nu = (3-1)(2-1) = 2; \quad \chi_2^2(10\%) \text{ c.v.} = 4.605$</p> <p>$3.95 < 4.605$ or not significant or do not reject H_0 (allow reject H_1)</p> <p>Insufficient evidence of an association between English and maths grades <u>or</u> there is support for the Director's belief <u>or</u> Student's grades in maths and English are independent</p> | 20 | 27.5 | 12.5 | 20 | 27.5 | 12.5 | <p>B1</p> <p>M1 A1</p> <p>M1, A1</p> <p>B1; B1</p> <p>M1</p> <p>A1 (9)</p> <p>B1 (1)</p> <p>10</p> |
| 20 | 27.5 | 12.5 | | | | | | |
| 20 | 27.5 | 12.5 | | | | | | |
| <p>(a)</p> | <p>1st B1 for both hypotheses in terms of independence or association and in context. Must mention Maths and English in at least one of the hypotheses. “relationship” or “correlation” or “connection” or “link” is B0</p> <p>1st M1 for some correct calculation seen</p> <p>1st A1 for all expected frequencies correct. Accept answers without formula seen.</p> <p>2nd M1 for some evidence seen of attempt to calculate test statistic. At least one correct term seen. Follow through their expected frequencies.</p> <p>2nd A1 for AWRT 3.95. Answers only please escalate!</p> <p>3rd M1 for correct comparison or statement – may be implied by correct conclusion.</p> <p>3rd A1 for conclusion in context using “association” or “independence” in connection with grades. Don't insist on seeing English or maths mentioned here. Use ISW for comments if a false statement and correct statement are seen.</p> | | | | | | | |
| <p>(b)</p> | <p>B1 If they just say expected frequencies are “small” they must go onto mention need to pool.</p> | | | | | | | |

| Question number | Scheme | Marks |
|-----------------|--|---|
| 3. | <p>$H_0 : \mu = 18, \quad H_1 : \mu < 18$</p> <p>$z = \frac{16.5 - 18}{\frac{3}{\sqrt{15}}} = -1.9364\dots$ AWRT – 1.94</p> <p>5% one tail c.v. is $z = (-) 1.6449$ or probability (AWRT 0.026) $(\pm) 1.6449$</p> <p>$-1.94 < -1.6449$ <u>or</u> significant <u>or</u> reject H_0 <u>or</u> in critical region</p> <p>There is evidence that the (mean) time to complete the puzzles has reduced</p> <p><u>Or</u> Robert is getting faster (at doing the puzzles)</p> | <p>B1, B1</p> <p>M1, A1</p> <p>B1</p> <p>M1</p> <p>A1f.t.</p> |
| | <p>1st & 2nd B1 must see and 18</p> <p>1st M1 for attempting test statistic, allow \pm. Or attempt at critical value for $\bar{X} : \mu - z \times \frac{3}{\sqrt{15}}$</p> <p>1st A1 for AWRT – 1.94. Allow use of $z = +1.94$ to score M1A1. Or critical value = AWRT 16.7.</p> <p>3rd B1 for AWRT 0.026 (i.e. correct probability only) or ± 1.6449. (May be seen in cv formula)</p> <p>2nd M1 for correct comparison or statement relating their test statistic and 1.6449 or their probability and 0.05. Ignore their hypotheses if any or assume they were correct.</p> <p>2nd A1f.t. for conclusion in context which refers to “speed” or “time”. Depends only on previous M</p> | <p>7</p> |

| Question number | Scheme | Marks | | | | | | | | | | | | | | | | | | | | | | | | |
|---------------------|--|----------------------|------|------|----------|---|----------|-------|----|----|----|----|----|-------|------|------|------|------|------|---------------------|------|------|------|------|------|--|
| 4. (a) | $\frac{0 \times 17 + 1 \times 31 + \dots}{17 + 31 + \dots} = \left(\frac{200}{100} = 2 \right), \quad \hat{p} = \frac{2}{20} = 0.1 \quad (\text{Accept } \frac{2}{20} \text{ or 2 per 20})$ | M1, A1 (2) | | | | | | | | | | | | | | | | | | | | | | | | |
| (b) | e.g. $r = 100 \times \binom{20}{2} (0.1)^2 (0.9)^{18}$ $r = 28.5, s = \text{AWRT } 9$ | M1 A1, A1 (3) | | | | | | | | | | | | | | | | | | | | | | | | |
| (c) | <table border="1" data-bbox="228 622 818 925"> <thead> <tr> <th>x</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>≥ 4</th> </tr> </thead> <tbody> <tr> <td>O_i</td> <td>17</td> <td>31</td> <td>19</td> <td>14</td> <td>19</td> </tr> <tr> <td>E_i</td> <td>12.2</td> <td>27.0</td> <td>28.5</td> <td>19.0</td> <td>13.3</td> </tr> <tr> <td>$\frac{(O-E)^2}{E}$</td> <td>1.89</td> <td>0.59</td> <td>3.17</td> <td>1.32</td> <td>2.44</td> </tr> </tbody> </table> $\sum \frac{(O-E)^2}{E} = \text{AWRT } 9.4$ | x | 0 | 1 | 2 | 3 | ≥ 4 | O_i | 17 | 31 | 19 | 14 | 19 | E_i | 12.2 | 27.0 | 28.5 | 19.0 | 13.3 | $\frac{(O-E)^2}{E}$ | 1.89 | 0.59 | 3.17 | 1.32 | 2.44 | Pooling M1 M1A1c.a.o. B1ft, B1ft |
| x | 0 | 1 | 2 | 3 | ≥ 4 | | | | | | | | | | | | | | | | | | | | | |
| O_i | 17 | 31 | 19 | 14 | 19 | | | | | | | | | | | | | | | | | | | | | |
| E_i | 12.2 | 27.0 | 28.5 | 19.0 | 13.3 | | | | | | | | | | | | | | | | | | | | | |
| $\frac{(O-E)^2}{E}$ | 1.89 | 0.59 | 3.17 | 1.32 | 2.44 | | | | | | | | | | | | | | | | | | | | | |
| (d) | $v = 5 - 2 = 3, \quad \chi_3^2(5\%) = 7.815$ H_0 : Binomial distribution is a good/suitable model/fit [Condone: B(20, 0.1) is...] H_1 : Binomial distribution is not a suitable model (Significant result) Binomial distribution is not a suitable model | both B1 A1cao (7) | | | | | | | | | | | | | | | | | | | | | | | | |
| (d) | defective items do <u>not</u> occur <u>independently</u> <u>or</u> <u>not</u> with <u>constant probability</u> | B1ft (1) | | | | | | | | | | | | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (a) | M1 for attempt to find mean or \hat{p} (as printed or better). The 0.1 must be seen in part (a). | | | | | | | | | | | | | | | | | | | | | | | | | |
| (b) | M1 for correct expression for r or s using the binomial distribution. Follow through their \hat{p} . | | | | | | | | | | | | | | | | | | | | | | | | | |
| (c) | 1 st M1 for some pooling (accept $x \geq 5$, obs.freq. ...14, 9, 10 and exp.freq. 19.0, s , 4.3) 2 nd M1 for calculation of test statistic (N.B. $x \geq 5$ gives 14.5). One correct term seen. 1 st B1ft for number of classes – 2 (N.B. $x \geq 5$ will have $6 - 2 = 4$) 2 nd B1ft for the appropriate tables value, ft their degrees of freedom. (NB $\chi_4^2(5\%) = 9.488$) 3 rd B1 (for hypotheses) allow just “ $X \sim B(20, 0.1)$ ” for null etc. 2 nd A1 for correctly rejecting Binomial model. No ft and depends on 2 nd M1. | | | | | | | | | | | | | | | | | | | | | | | | | |
| (d) | B1ft for independence or constant probability – must mention defective items or defectives Follow through their conclusion in (c). So if they do not reject they may say “defectives occur with probability 0.1”. Stating the value implies constant probability. | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question number | Scheme | Marks |
|-----------------|---|--|
| 5. (a) | $\hat{\mu} = \bar{x} = \frac{361.6}{80}, = \underline{4.52}$ $\hat{\sigma}^2 = s^2 = \frac{1753.95 - 80 \times \bar{x}^2}{79} = (1.51288...)$ <p style="text-align: right;">AWRT <u>1.51</u></p> | M1, A1 M1A1ft A1 (5) |
| (b) | $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A > \mu_B$ <p style="text-align: right;">Denominator</p> $z = \frac{4.52 - 4.06}{\sqrt{\frac{1.51...}{80} + \frac{2.50}{60}}} = \left(\frac{0.46}{\sqrt{0.0605...}} \right)$ <p style="text-align: right;">z</p> $= (+) 1.8689... \quad \text{AWRT } (+) \underline{1.87}$ <p>One tail c.v. is $z = 1.6449$ (AWRT 1.645 or probability AWRT 0.0307 or 0.0308)</p> <p>(significant) there is evidence that diet A is better than diet B <u>or</u> evidence that (mean) weight lost in first week using diet A is more than with B</p> | B1 B1 M1 dM1 A1 B1 A1ft (7) |
| (c) | CLT enables you to assume that \bar{A} and \bar{B} are normally distributed | B1 (1) |
| (d) | Assumed $\sigma_A^2 = s_A^2$ and $\sigma_B^2 = s_B^2$ (either) | B1 (1) |
| 14 | | |
| (a) | <p>2nd M1 for a correct attempt at s or s^2, A1ft for correct expression for s^2, ft their mean.</p> <p>N.B. $\sigma^2_n = 1.49... \text{ so } \frac{80}{79} \times 1.49... \text{ is M1A1ft}$</p> | |
| (b) | <p>1st B1 can be given for $\mu_1 = \mu_2$, but 2nd B1 must specify which is A or B.</p> <p>1st M1 for the denominator, follow through their 1.51.</p> <p>Must have square root can condone 2.50^2 but $\sqrt{\frac{1.51^2}{80} + \frac{2.50^2}{60}}$ is M0.</p> <p>Allow $\sqrt{\frac{1.51}{79} + \frac{2.50}{59}}$ leading to AWRT 1.85 to score M1M1A0 in (b) and can score in (d).</p> <p>2nd dM1 for attempting the correct test statistic, dependent on denominator mark</p> <p>1st A1 for AWRT ± 1.87, may be implied by a correct probability.</p> <p>2nd A1ft ft their test statistic vs their cv only if H_1 is correct and both Ms are scored</p> | |
| (c) | B1 for stating <u>either</u> \bar{A} or \bar{B} (but not A or B) are normally distributed | |
| (d) | B1 for either, can be stated in words in terms of variances or standard deviations. | |

| Question number | Scheme | Marks |
|--|---|---|
| 6. | $\bar{x} = \frac{1}{2}(123.5 + 154.7) = \underline{139.1}$ <p style="text-align: right;">2.5758</p> <p>"their 2.5758" $\frac{\sigma}{\sqrt{n}} = 154.7 - 139.1 = 15.6$</p> <p style="text-align: right;">AWRT 1.96</p> <p>"their 1.96" $\frac{\sigma}{\sqrt{n}} = \frac{15.6 \times 1.96}{2.5758} = (11.87\dots)$</p> <p>So 95% C.I. = $139.1 \pm 11.87\dots = (127.22\dots, 150.97\dots)$ AWRT <u>(127, 151)</u></p> | <p>B1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> |
| 6 | | |
| <p>1st B1 for mean = 139.1 only</p> <p>1st M1 for UL – mean or mean – LL set equal to z value times standard error or some equivalent expression for standard error. Follow through their 2.5758 provided a z value.</p> <p>May be implied by $\frac{\sigma}{\sqrt{n}} = 6.056\dots$ [N.B. $\frac{15.6}{2.3263} = 6.705\dots$]</p> <p>Condone poor notation for standard error if it is being used correctly to find CI.</p> <p>2nd M1 for full method for semi-width (or width) of 95% interval</p> <p>Follow through their z values for both M marks</p> <p>N.B. Use of 2.60 instead of 2.5758 should just lose 2nd B1 since it leads to AWRT (127, 151)</p> | | |

| Question number | Scheme | Marks |
|-----------------|---|---|
| 7. (a) | <p>Let $X = L - 4S$ then $E(X) = 19.7 - 4 \times 4.9 = 0.1$ $\text{Var}(X) = \text{Var}(L) + 4^2 \text{Var}(S) = 0.5^2 + 16 \times 0.2^2$ $= 0.89$ $P(X > 0) = [P(Z > -0.10599\dots)]$ $=$ AWRT <u>(0.542 – 0.544)</u></p> <p>(b) $T = S_1 + S_2 + S_3 + S_4$ (May be implied by 0.16) $E(T) = 19.6$ $T \sim N(19.6, 0.16)$ $\text{Var}(T) = 0.16$ or 0.4^2</p> <p>(c) Let $Y = L - T$ $E(Y) = E(L) - E(T) = [0.1]$ $\text{Var}(Y) = \text{Var}(L) + \text{Var}(T) = [0.41]$ Require $P(-0.1 < Y < 0.1)$ $= P(Z < 0) - P(Z < -0.31\dots)$ or $0.5 - P(Z < -0.31\dots)$ or $P(Z < 0.31\dots) - P(Z < 0)$ $= 0.1217$ (tables) or $0.1226\dots$ (calc) AWRT <u>(0.122 – 0.123)</u></p> | <p>M1, A1 M1, M1 A1 M1 A1 (7)</p> <p>M1 B1 A1 (3)</p> <p>M1 M1 M1 M1 A1 (5)</p> <p style="text-align: right;">15</p> |
| (a) | <p>1st M1 for defining X and attempting $E(X)$ 1st A1 for 0.1. Answer only will score both marks. 2nd M1 for $\text{Var}(L) + \dots$ 3rd M1 for $\dots 4^2 \text{Var}(S)$. For those who don't attempt $L - 4S$ this will be their only mark in (a). 2nd A1 for 0.89 4th M1 for attempting a correct probability, correct expression and attempt to find, which should involve some standardisation: ft their $\sqrt{0.89}$ and their 0.1. If 0.1 is used for $E(X)$ answer should be > 0.5, otherwise M0.</p> | |
| (c) | <p>1st M1 for a correct method for $E(Y)$, ft their $E(T)$. 2nd M1 for a correct method for $\text{Var}(Y)$, ft their $\text{Var}(T)$. Must have +. 3rd M1 for dealing with the modulus and a correct probability statement. Must be modulus free. May be implied by e.g. $P(Z < \frac{0.2}{\sqrt{\text{their } 0.41}}) - 0.5$, or seeing both 0.378... (or 0.622...) <u>and</u> 0.5 4th M1 for correct expression for the correct probability, as printed or better. E.g. $0.5 + 0.378\dots$ is M0 A1 for AWRT in range.</p> | |